

# Towards enhanced regionalization of hydrologic-hydraulic river network models with assimilation of multi-source data and SWOT hydraulic visibility

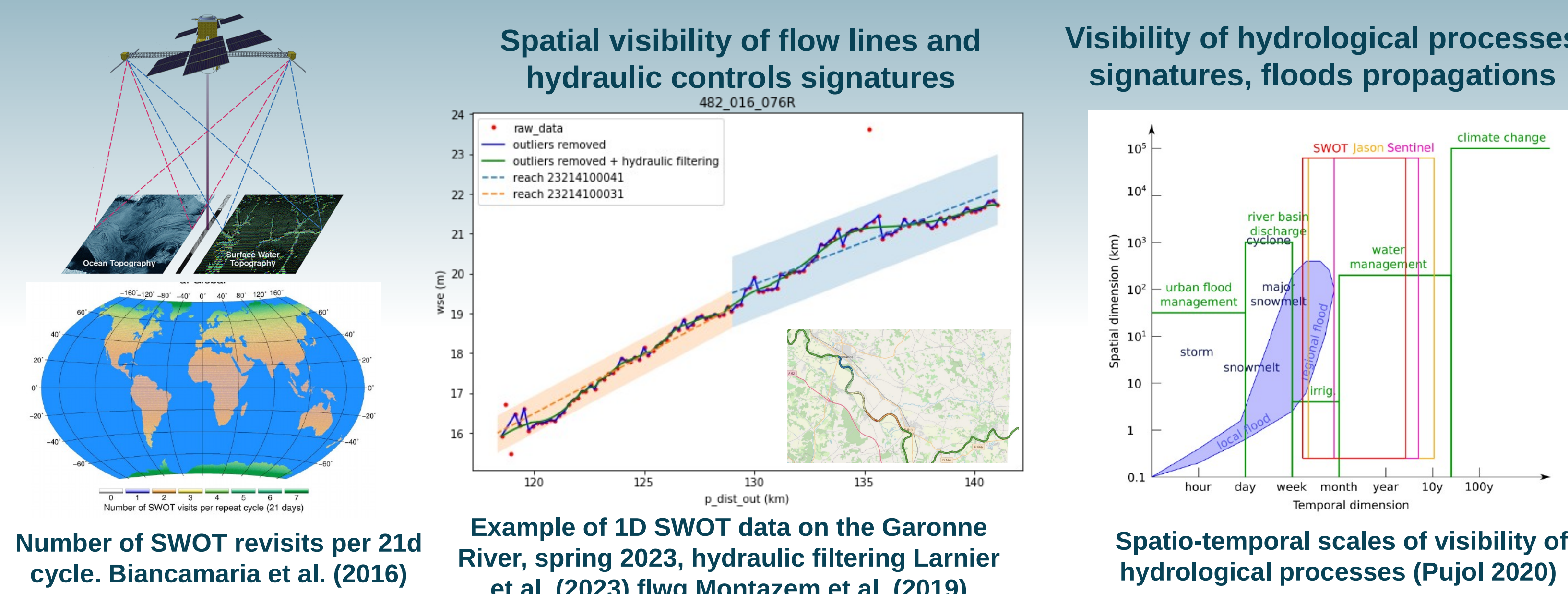
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MathHydroNum project team website :  
<https://mathhydronum.insa-toulouse.fr/>  
DassHydro open source softwares on GitHub  
<https://github.com/orgs/DassHydro>  
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## Context: worldwide hydraulic visibility of rivers surfaces variabilities with SWOT



## Challenges in Hydrology from WS signatures (SWOT, multi-mission altimetry, optical/radar water extents, in situ) :

- Estimation of rivers discharge (plus unobserved and uncertain bathymetry-friction) from water surface observables is an ill-posed hydraulic inverse problem – local (at a section) plus spatial equifinality (Garambois-Monnier 2015, Larnier-Monnier-Garambois 2020, Garambois et al. 2020)
- Uncertainty (quantification) reduction by data assimilation of WS obs. (w/wo in situ data) in river networks-floodplains hydraulic models, feedback to hydrological model is faced with data uncertainties, heterogeneity in nature and spatio-temporal sampling – SWOT temporal sparsity wrt higher hydrological frequencies (Brisset et al. 2016-18, Pujol et al. 2020)
- Information feedback from observations of hydraulic network-floodplains to hydrology ; Regionalization of models parameters and learning of physical laws from massive data from reach to global scale

## Outline

- Differentiable spatialized hydrological - hydraulic modeling at basin scale**, aiming to (i) transfer hydrological fluxes and states consistently, while (ii) performing spatialized info feedback from heterogeneous data to ungauged reaches/basins.
- Model building and DA from multi-source data** – spatially dense water surface (WS) elevation and width (dynamic), sparse but constraining discharge data.
- Variational data assimilation (VDA) of multi-mission altimetry** - Optimization of spatially distributed model parameters (bathymetry-friction, inflows ~ high dimension)
- Preparing use of SWOT 1 day orbit data** on the Maroni River (French Guyana)

## The differentiable hydrologic-hydraulic model

Spatially distributed hydrological model applied to a basin  $\Omega_r$  such that  $\forall x' \in \Omega_r, t > 0$ :

$$\mathcal{M}_{rr} : \begin{cases} \frac{d}{dt} \mathbf{h}(x', t) = f(\mathbf{h}; \mathbf{I}, \mathbf{D}; \mathbf{h}_0, \theta_{rr}; x', t) \\ Q(x', t) = g(\mathbf{h}; \mathbf{I}, \mathbf{D}; \mathbf{h}_0, \theta_{rr}; x', t) \end{cases}, \forall x' \in \Omega_r, t \in ]0, T]$$

With  $\mathbf{h}$  the model state vector,  $\mathbf{I}$  and  $\mathbf{D}$  respectively the spatio-temporal atmospheric inputs fields and basin physical descriptors ;  $\mathbf{h}_0$  the initial state,  $\theta_{rr}$  the hydrological control vector and  $\mathbf{h}_0 = \mathbf{h}(t=0)$  the initial state ;  $Q$  the spatio temporal output discharge.

**1D full Saint-Venant hydraulic model over the network  $\Omega_{hy}$**  (connected closed lines), such that  $\forall x \in \Omega_{hy}, t > 0$ :

$$\mathcal{M}_{hy} : \begin{cases} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \\ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) = -gA \left( \frac{\partial Z}{\partial x} - S_f \right) + U \delta_l q_l \end{cases}$$

With  $A$  the wetted area,  $Q$  the discharge,  $q_l$  the lateral inflow,  $Z$  the WS elevation such that  $Z = b + h$  with  $b$  the river bed elevation and  $h$  the flow depth,  $S_f = Q^2 / K^2 A^2 R_n^{4/3}$  the friction source term. Denote by  $\theta_r$  the hydraulic control.

**The weakly coupled hydrological-hydraulic model**, obtained via hydrological state-flux remapping onto the hydraulic model (inflows here), writes as :

$$(A, Q)(x, t) = \mathcal{M}(\mathbf{I}, \mathbf{D})(x', t), \quad x' \in \Omega_{hy}, \quad x \in \Omega_{rr}$$

with  $\mathcal{M} = \mathcal{M}_{hy}(\theta_{hy}) \circ \mathcal{M}_{rr}(\theta_{rr})$

**The composed adjoint model reads:**

$$(D_{\theta_{rr}} \mathcal{M}_{rr})^T(\theta_{rr}) \circ (D_{\theta_{hy}} \mathcal{M}_{hy})^T(\theta_{hy})$$

**The full control vector is:**  $\theta = (\theta_{rr}, \mathbf{h}_0, \theta_{hy})^T$

## The adjoint based variational data assimilation algorithm

Heterogeneous WS width-elevation observation set:

$$Y^* := \left\{ \left( Z^*(x_{oz}=1..N_z, t_{pz}=1..P_z(oz)) \right); W^*(x_{ow}=1..N_w, t_{pw}=1..P_w(ow)) \right\}$$

$N_z$  spatial resp.  $P_z$  temporal points

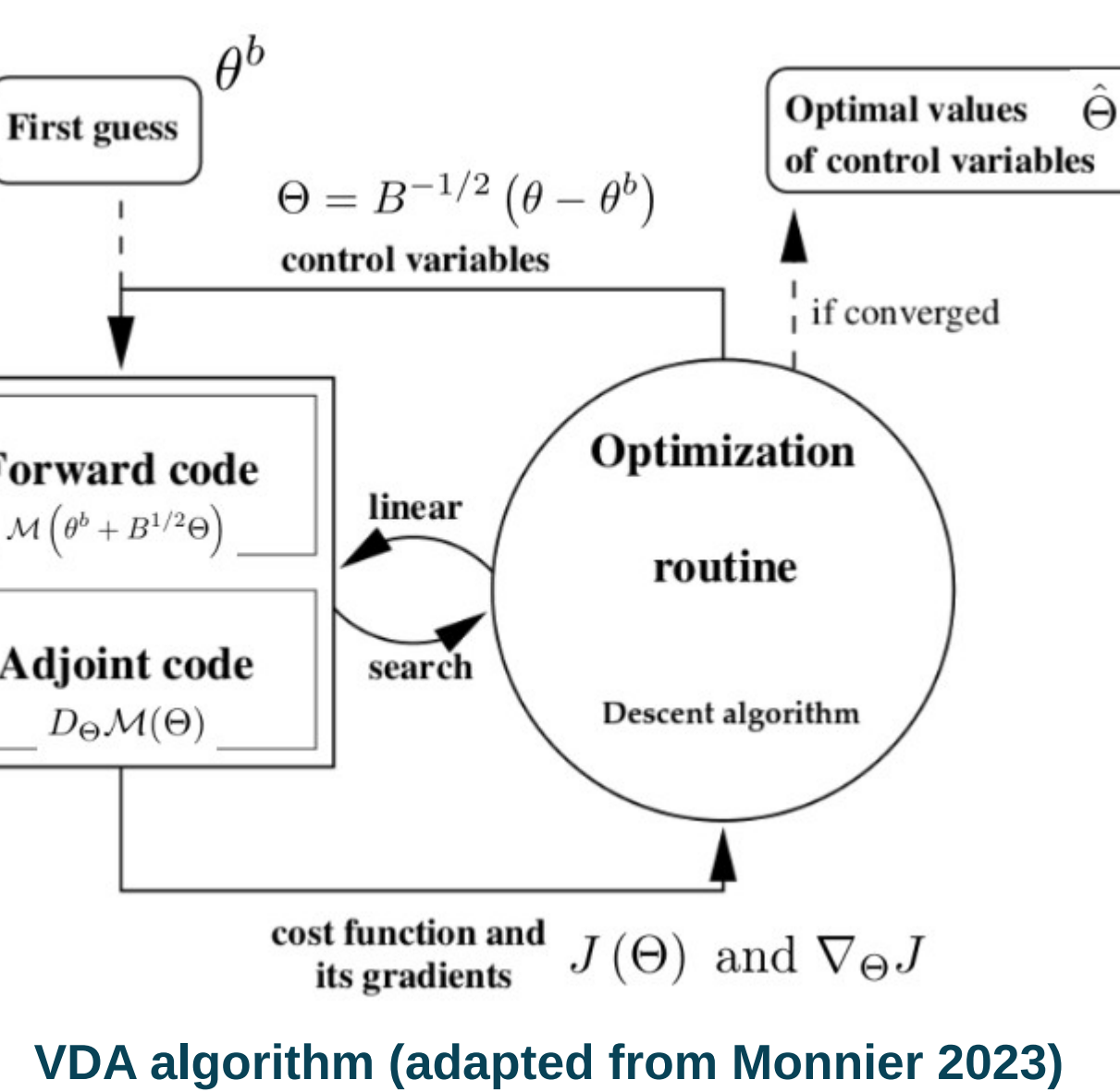
The cost function measuring the model - observation discrepancy is defined as:

$$J(\Theta) = \frac{1}{2} \left\| H(\theta^b + B^{1/2} \Theta) - Y^* \right\|^2$$

With variable change used to constrain ill-posed inverse problem (cf. Larnier et al 2020) :

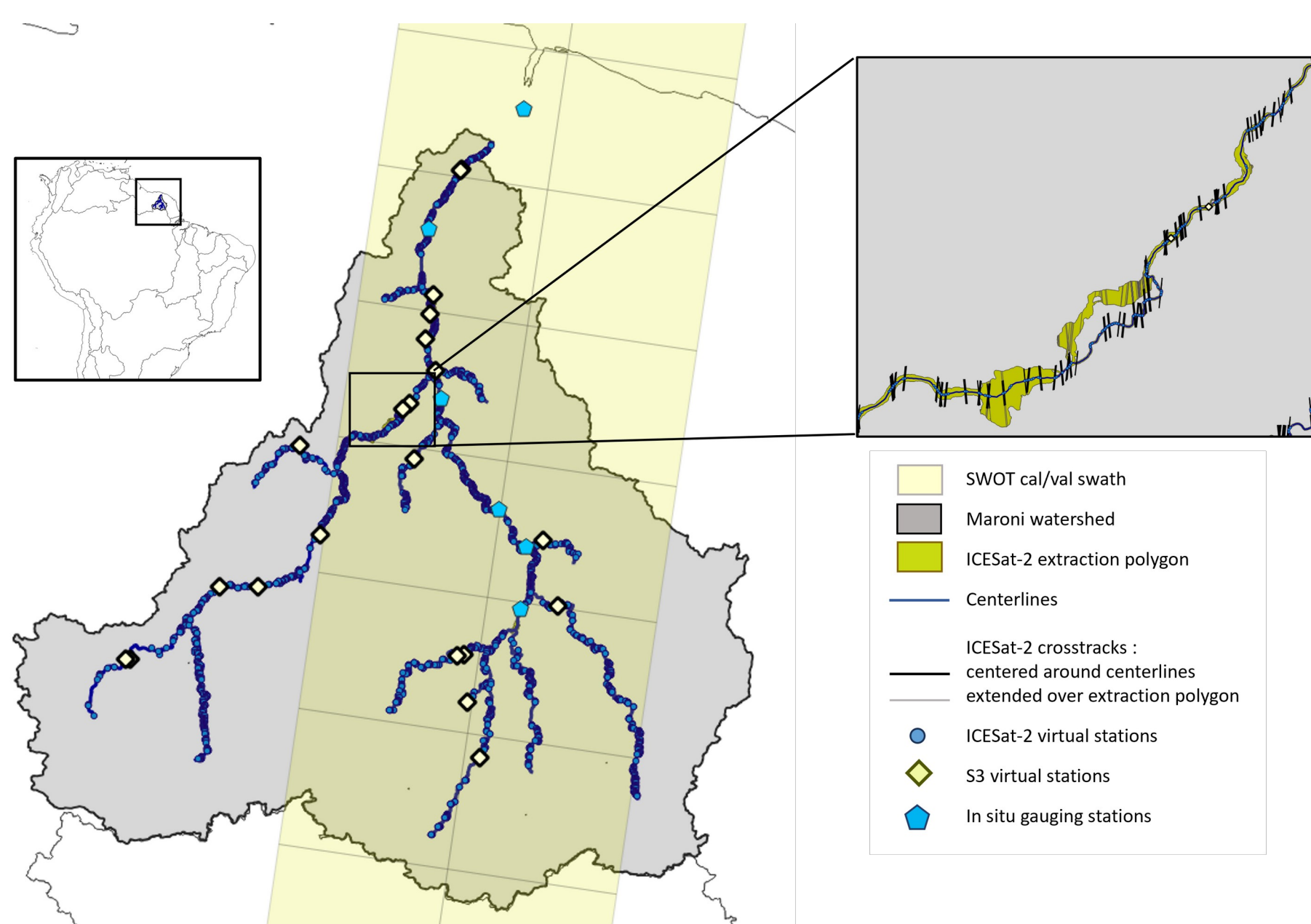
$$\Theta = B^{-1/2} (\theta - \theta^b)$$

**Optimization problem solved:**  $\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} J(\Theta)$

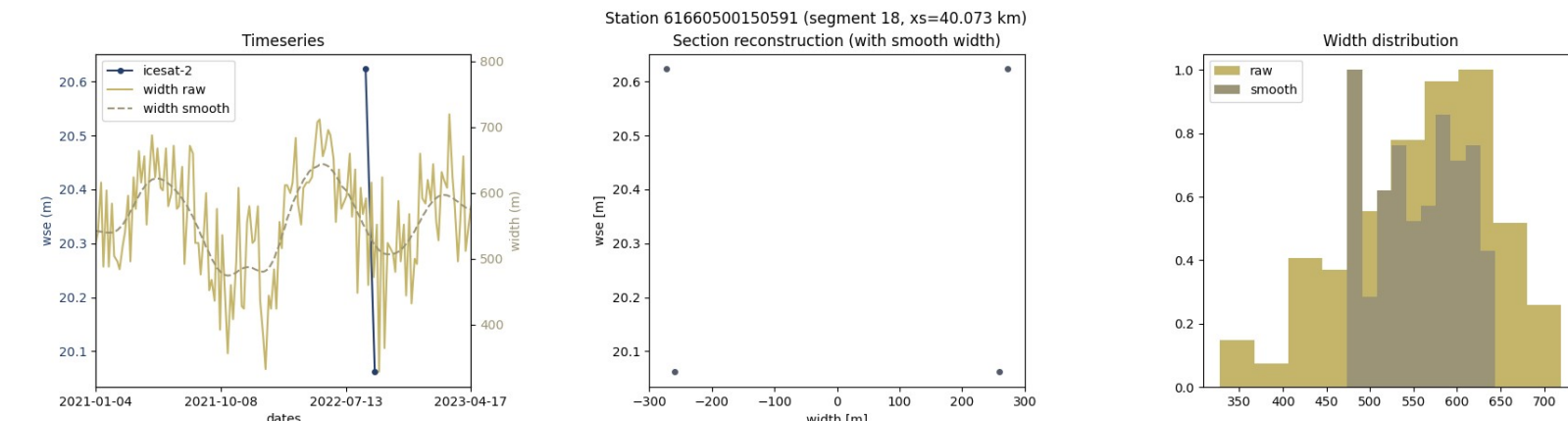


VDA algorithm (adapted from Monnier 2023)

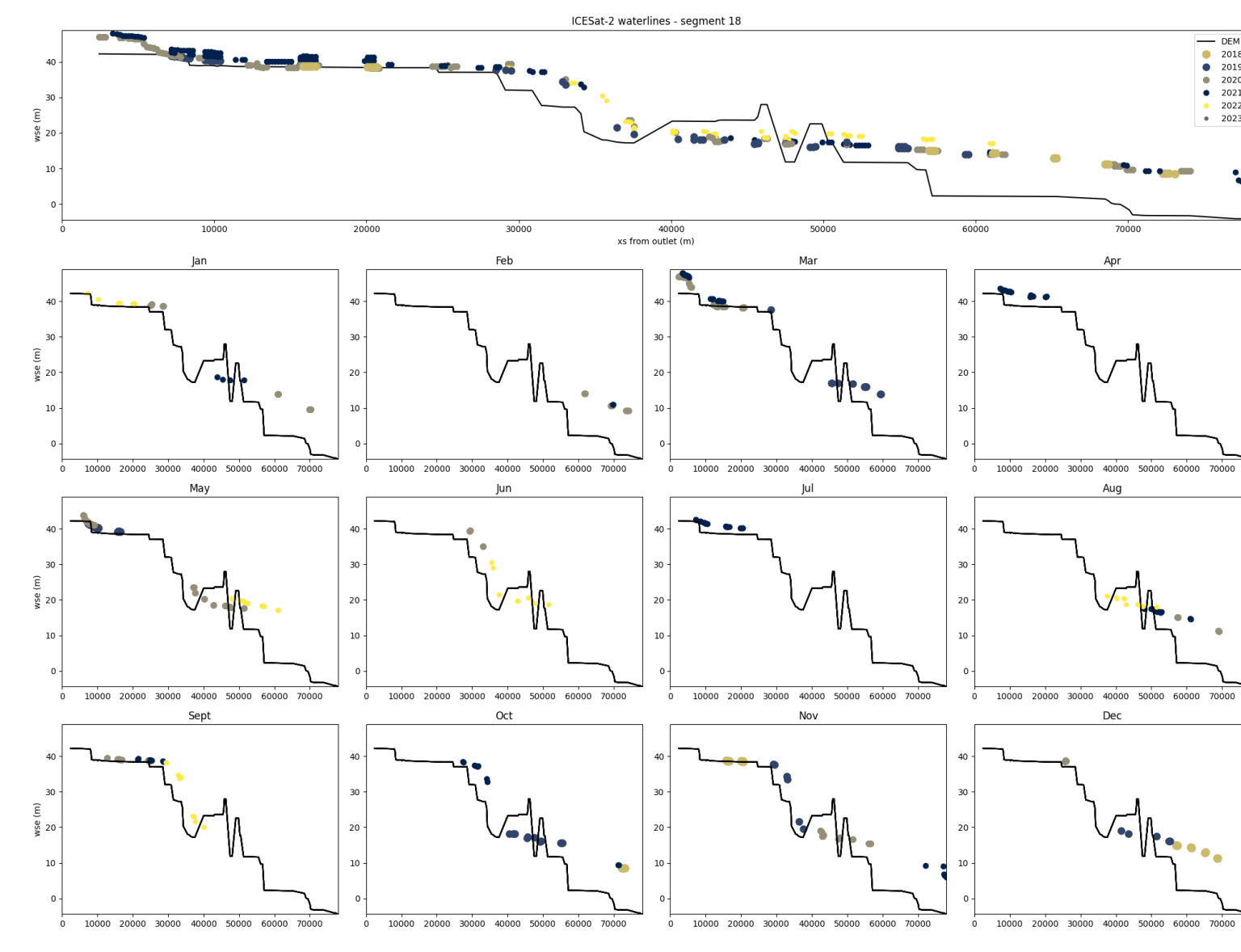
## Model building from multi-source data over the Maroni basin



Hydraulic network (in blue) simulated with DassFlow 1D inflowed by MGB hydrological model (Paiva et al. 2013).



Example of Sentinel water extent time series



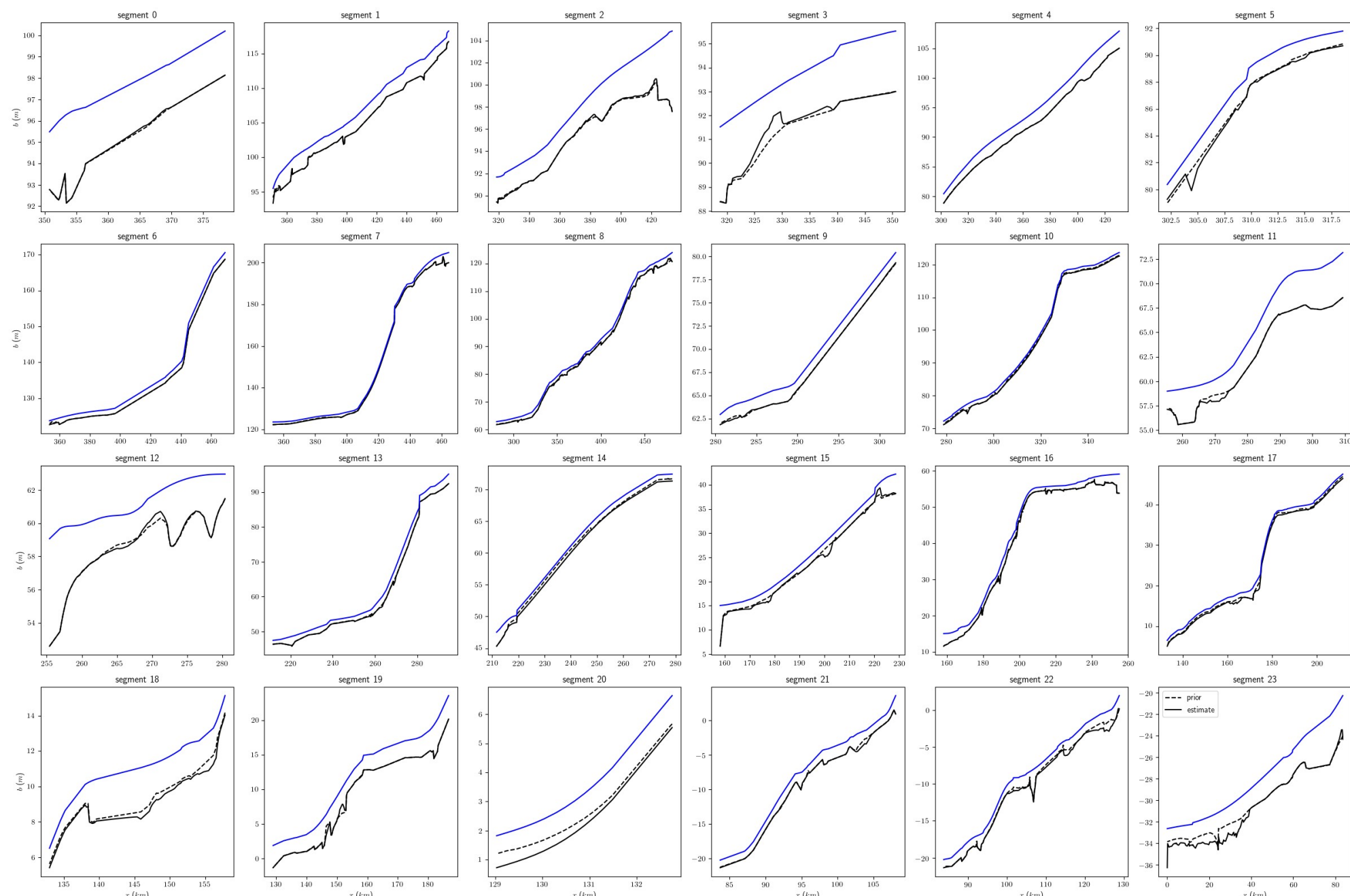
Icesat2 data "climatology over the studied Maroni network"

- Automatic processing chain for Icesat WSE from raw data, for extracting WS width from water mask (dynamic river extent)
- Automatic coupling algorithm with inflow points determination
- River channels wet bathymetry shape inferred from WS data with (i) prior friction value, (ii) backwater curve solved over the network given hydrological inflows.

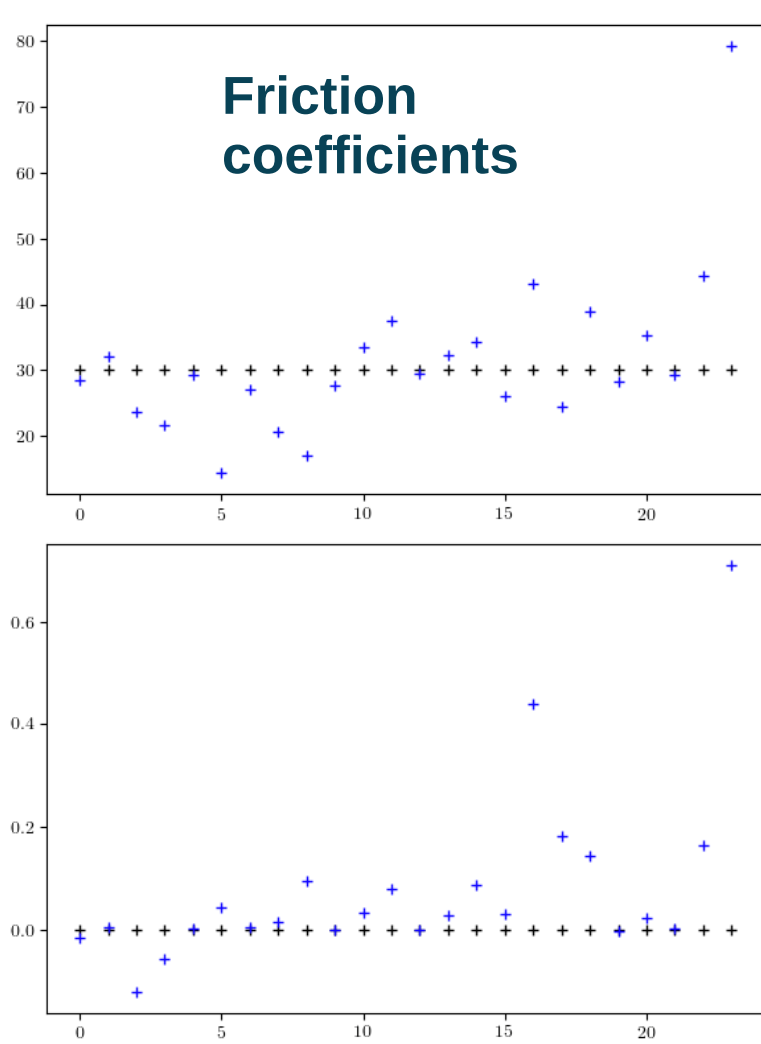
## Multi-mission altimetry VDA results on the Maroni River

Hydraulic parameter vector inferred from remaining IceSat & Sentinel WSE:

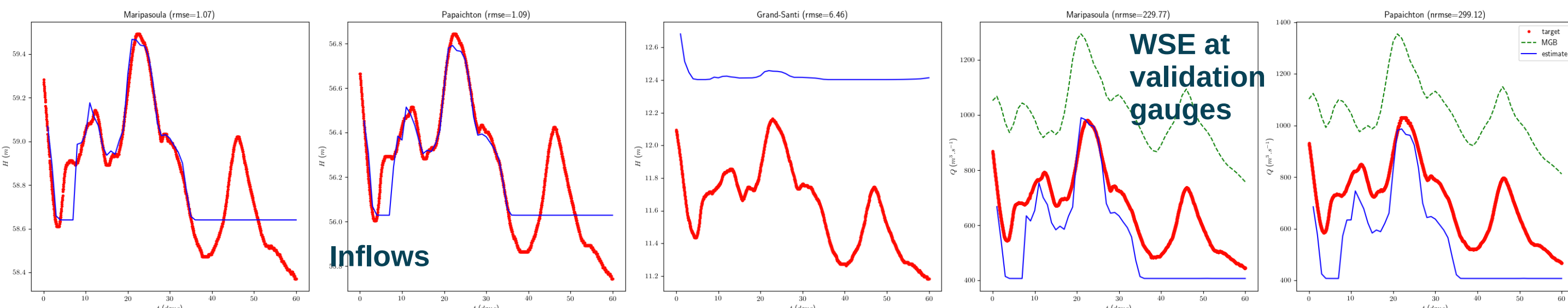
$$\theta := \theta_{hy} = \left[ \left( Q_{in,u}^0, \dots, Q_{in,u}^{P(u)} \right)_{u=1..N_{inflow}}; \left( b_{1,s}, \dots, b_{N_b(s),s} \right)_{s=1..N_{seg}}; \left( K_{1,s}, \dots, K_{N_K(s),s} \right)_{s=1..N_{seg}} \right]^T$$



Inferred bathymetry



Friction coefficients

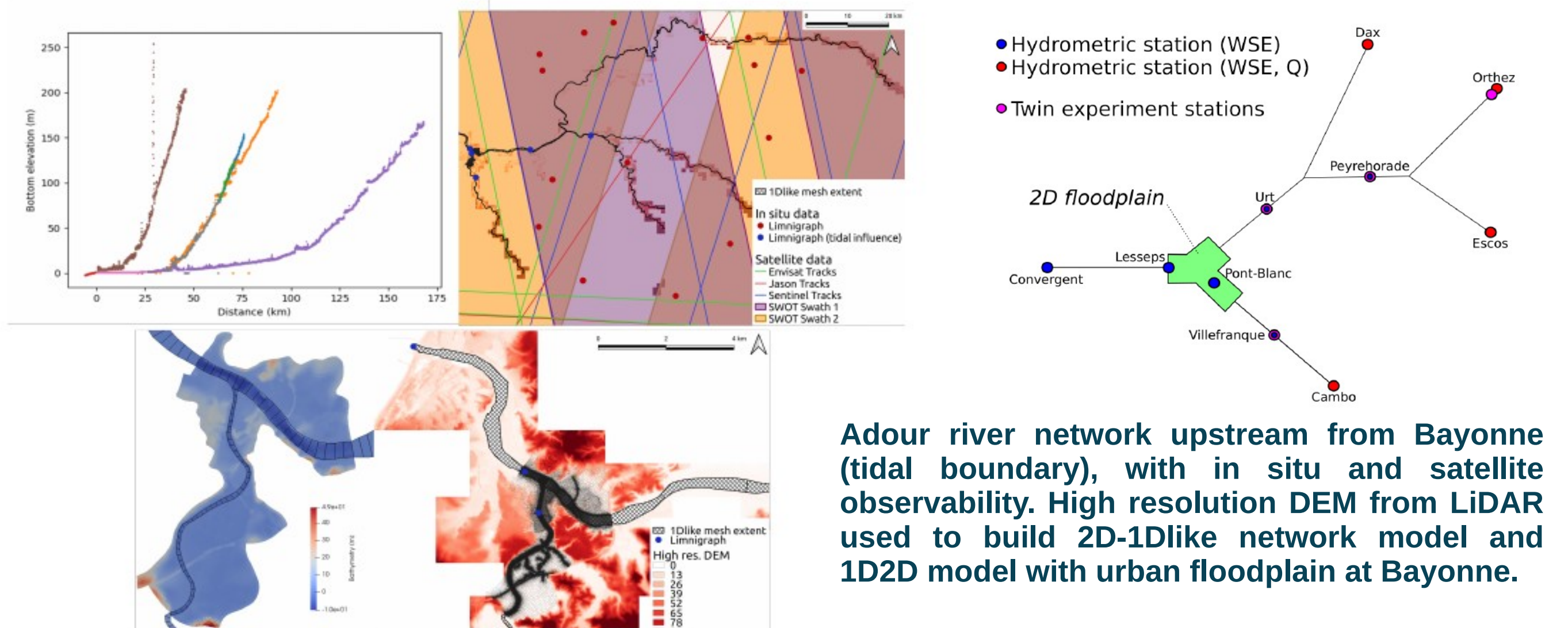


Inflows

WSE at validation gauges

## 2D-1D river network model

Multi-dimensional (multi-D) differential hydraulic-hydrological numerical model with VDA capabilities (Pujol et al. 2022) applied on the Adour basin



Adour river network upstream from Bayonne (tidal boundary), with in situ and satellite observability. High resolution DEM from LiDAR used to build 2D-1Dlike network model and 1D2D model with urban floodplain at Bayonne.

## Conclusions and perspectives :

- VDA applied over full river network hydraulic modeling with heterogeneous data; extension to floodplain observations possible.
- Ongoing study of inference capabilities (with SWOT data too) - effective bathymetry-friction, discharge; expectable accuracy and space-time resolution?
- Ongoing coupling with SMASH distributed model and differentiation of the full chain, enabling hybrid regionalization with algorithm from Huynh et al. (2023), testing information feedback (cf. Pujol et al. 2022, Garambois et al. 2023 SHF talk) to spatially learnable hydrological solvers (Huynh et al. In prep).

**Our main references related to discharge inference with SWOT, models.** On equifinality issues : [Garambois-Monnier, AWR'15], [Larnier-Monnier-Garambois et al., IPSE'20]. On identifiability capabilities : [Brisset-Monnier-Garambois et al., AWR'18]. On accurate flow solvers : [Monnier-Couderc et al., AWR'16]. On algorithm evaluations : [Tuozzolo et al. GRL'19], [Frasson et al., WRR'21]. Latest version of HiVDI and codes : [Larnier-Monnier, Comput. GeoSc.'23], [Pujol et al., GMD'22]. Model learnt Stage-Fall-Discharge laws : [Malou et al. JOH 2022]. Physical kernels for VDA: [Malou-Monnier, InvPB, 2022]. On complex flows investigations - applications : [Garambois et al. Hydro. Proc'17], [Garambois et al. JoH'20], [Pujol et al., JoH'20]. On SWOT data segmentation-filtering : [Montazem et al., GRL'19]